Comparison between linear and nonlinear control Strategies for Induction Motor

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Abstract
This paper presents a comparison of two different control strategies for Squirrel-Cage Induction Motor (IM): conventional Proportional-Integral (PI) controller and modern Sliding Mode Controller (SMC). Two control laws are proposed to control speed and rotor flux independently. Convergence analysis of the SMC, based on Lyapunov theory, is proved. Both strategies are investigated and simulated using MATLAB/Simulink under nominal case. Moreover, the robustness of both methods with respect to the parameters uncertainties is verified by significant simulation results.

The study shows that nonlinear control strategy based Sliding mode technique possesses not only high-performance dynamic characteristics, but also robustness with respect to parameter variations and external load disturbances compared to PI controller.

Keywords: Field oriented control, linear control, Non-linear control, PI controller, sliding mode controller, Speed controller, and Induction motor.

List of symbols
B  Coefficient of viscous friction (Nm/s/rad).
F  Supply frequency (Hz).
\(I_{d0}, I_{q0}\) d-and q- axis stator currents (A).
J  Moment of inertia (kg.m^2).
Kt  Torque constant.
\(L_d, L_q, L_m\) Stator self, rotor self, and mutual inductances (H).
P  Number of poles pairs.
R_s, R_r  Stator and rotor resistances (\(\Omega\)).
\(T_e, T_l\) Rotor time constant (s).
\(V_{ds}, V_{qs}\) d-and q- axis stator voltages (V).
\(\omega_s, \omega_r, \omega_i\) Synchronous, rotor, slip speed (rad/sec).
\(\lambda_{ds}, \lambda_{qs}\) d-and q- axis stator fluxes (Wb).
\(\lambda_{dr}, \lambda_{qr}\) d-and q- axis rotor fluxes (Wb).

I. INTRODUCTION
Induction motors are being applied today to a wider range of applications requiring variable speed, since relatively inexpensive and rugged machines especially squirrel cage type because it can be built without slip rings or commutators. The control of induction motors is a challenging problem since it has a nonlinear model, rotor variables are rarely measurable, its parameters vary with operating conditions, and the coupling between the different variables of control.

Several techniques are used to control the induction motor. These techniques can be classified as Scalar control or Volts/Hertz (V/f), Field oriented control (FOC) or Vector Control, and Direct Torque control (DTC). For the last three decades most of drives have employed vector control methods thanks to their advantage compared to other methods (V/f) and (DTC). The disadvantage of those controllers is they only consider the motor model steady state. Consequently, a controller based on these methods cannot achieve the ultimate performance during transients [1]. The classical DTC has other drawbacks such as variable switching frequency, high torque, and flux ripples. These drawbacks degrade its performance, especially at low speed operation [2], [3]. A new SMC approach was
investigated in [4] to solve the DTC problems for PMSM. The FOC technique has been widely used for high-performance induction motor drives. It achieves effective decoupling between torque and flux, which allows control of speed and torque with high static and dynamic performances [5], [6]. Classical PI controller is a simple linear method used in control of induction motor drives. However, the main drawbacks of PI controller are the sensitivity of performance to the system-parameter variations and inadequate rejection of external disturbances and load changes, and un-modeled and nonlinear dynamics [7]. There are two ways to solve these problems. The first is to perform on-line identification of the motor parameters and accordingly update the values used in the controller. Many online identification schemes have been designed [8]-[10]. These methods have provided some improvements, but they are quite complex because they either require more parameters or have hardware complications. The other solution is to use a robust control algorithm insensitive to the motor parameter variations. Therefore, many studies have been made on the motor drives in order to preserve the performance under these parameter variations and external load disturbances, such as nonlinear control i.e. (classical sliding mode control, high order sliding mode control, back-stepping control...), adaptive control, and non-model control i.e. (neural control, fuzzy control, and genetic algorithm) [11]-[15]. The design of fuzzy control (FC) has relied on trial and error. In practice, a precise knowledge of the plant of the complex systems is often difficult to obtain. So an efficient FC cannot be achieved [16]. In recent years, to tackle this problem, the neural networks become attractive approach. Generally, this control strategy is known as neuro-fuzzy controllers (NFCs). A NFC is suitable for control of systems consisting of uncertainties and nonlinearities. Although the NFC approaches can also achieve self-learning, they are inappropriate for on-line learning real-time control, since the learning process is time-consuming [17]-[19]. The fuzzy rules number will influence on the controller accuracy and implementation consideration.

To overcome the above system uncertainties, the SMC has been focused on many studies and research for the control of the AC servo drive system in the past decade [20]-[22]. The sliding-mode control can offer many good properties, such as good performance against un-modeled dynamics, insensitivity to parameter variations, external disturbance rejection and fast dynamic response [23]. The main problem of SMC is the chattering phenomenon which occurs due to the high gain values.

This paper is organized as follows. Section II introduces the mathematical model of IM. Section III, presents a PI controller. Section IV, a SM controller is designed and its stability analysis is shown. In section V, the simulation results are presented for the nominal case and different cases to test the robustness of the system against load application and parameters variations. Finally, section VI gives some conclusions and future work.

II. Induction Motor Modeling

The induction machine considered in this work has a three-phase stator and a squirrel cage rotor. Under the classical assumptions [25], the IM model in d-q reference frame can be represented as follows:

$$\begin{bmatrix} \frac{d\theta}{dt} \\ \frac{d\omega}{dt} \\ \frac{dV_{ds}}{dt} \\ \frac{dV_{qs}}{dt} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & 0 & 0 \\ -a_2 & a_1 & 0 & 0 \\ a_2 & a_1 & a_2 & -1 \\ 0 & 0 & 0 & a_1 \end{bmatrix} \begin{bmatrix} u_d \\ u_q \\ V_{ds} \\ V_{qs} \end{bmatrix} + \begin{bmatrix} \frac{1}{l_d} \\ \frac{1}{l_q} \end{bmatrix}$$

(1)

Where:

- \( a_1 = \frac{1}{\sigma l_d} (R_s + R_r \frac{R_r}{sL_m}) \)
- \( a_2 = \frac{1}{\sigma l_d} (\frac{R_r}{sL_m}) \)
- \( a_3 = \frac{R_r}{\sigma l_d} \)
- \( \sigma = 1 - \frac{1}{\frac{1}{l_d} + \frac{1}{l_q}} \) = leakage coefficient and
- \( \omega_s = \omega_r - P \omega_f \) = Slip frequency.

The developed electromagnetic torque of the IM can be expressed as:

$$T_e = \frac{3}{2} \frac{R_r}{l_d} \omega_s (\lambda_{dq} \dot{i}_{q} - \lambda_{d} \dot{i}_{d})$$

(2)

The mechanical equation of IM:

$$\omega_r = \frac{1}{J} (T_e - B\omega_r - T_f)$$

(3)

Where: \( T_e \) and \( T_f \) are the electromagnetic and load torque respectively.

III. PI Control Design

Since the PI controllers are widely used in induction motor drives and proved to be simple and effective in steady state operation. A PI controller is designed to control the speed of induction motor as illustrated in Fig.1. Based on the vector control principle, the flux
component (d-axis component) of stator current (i_{d*}) is aligned with the direction of rotor flux (\lambda_r) and the torque component of stator current (i_{q*}) is aligned with the perpendicular direction to it, under this condition:
\lambda_{r*} = 0, \lambda_{q*} = 0, and \lambda_{d*} = \lambda_r = \text{rotor flux}

(4) hence, the developed electromagnetic torque is given by:

(5) T_e = \frac{3}{2} l_m l_e (\lambda_{d*} \omega_{q*}) = K_t \lambda_{d*} \omega_{q*}

With K_t = \frac{3}{2} l_m l_e

Considering the field orientation condition, the dynamic model of the induction motor becomes:

\[
\begin{pmatrix}
\dot{i}_{d*} \\
\dot{i}_{q*} \\
\dot{\lambda}_{d*} \\
\dot{\lambda}_{q*}
\end{pmatrix} =
\begin{bmatrix}
-a_1 & -a_2 & -\alpha & 0 & 1 & 0 & 0 \\
-a_2 & -a_3 & 0 & -\alpha & 0 & 1 & 0 \\
0 & 0 & -\omega_e & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -\omega_e & 1 & 0 & 0
\end{bmatrix}
\begin{pmatrix}
i_{d*} \\
i_{q*} \\
\lambda_{d*} \\
\lambda_{q*}
\end{pmatrix}
\]

(6)

From (6), the following relations can be obtained:

(7) \lambda_{d*} = \frac{i_{q*}}{\omega_e i_{q*}}

(8) \omega_{q*} = \frac{i_{q*}}{\tau_{L_d} e_{d*}}

Where \tau = L / R is the rotor electrical time constant. In the steady state, \lambda_{d*} = \lambda_{q*} = 0.

As depicted in Fig. 1, the controller consists of three PI loops to control three interactive variables independently. The rotor speed, rotor flux and rotor torque are each controlled by a separated PI controller and a decoupling between the speed and flux loop is done. The PI controller gains can be obtained by tuning the PI controllers to obtain less error in the system [24].

1. Speed PI regulator

The objective of this regulator is to track a speed reference. Its input is the reference speed (\omega_r) and the measured speed (\omega_e). Its output is the reference torque (T_e*).

From (3), the following transfer function can be found

(10) \frac{\omega_e}{\omega_r} = \frac{k_{\omega_e}}{s + \tau_{\omega_e}}

Where:
k_{\omega_e} = \frac{1}{\tau_{\omega_e}} is the static gain and
\tau_{\omega_e} = \frac{1}{\tau_{\omega_e}} is mechanical time constant.

2. Currents PI regulator

The machine equations used for the formulation of the control scheme are not linear between the inputs and outputs. So input-output linearization technique is applied such that the relationship between the input and output variable is linear after some feedback compensation then the PI controller can be used.

IV. SLIDING MODE CONTROL DESIGN

This section presents the design of SMC for IM. The control objective is to control the speed of the motor to follow a reference trajectory, and maintain the rotor flux as constant and oriented to the d-axis of the synchronous reference frame. The sliding mode is a technique to adjust feedback by previously defining a surface. The controlled system will be forced to that surface, then the behavior of the system slides to the desired equilibrium point.

1. Speed Sliding Mode Control

The objective is to develop a control law in order to the speed of the motor tracks a desired reference i.e. (\omega_r = \omega_r^*) in this case the reference is \omega_r^* and the controller is \omega_e.

The error of the speed is defined as:

(11) \theta = \omega_e - \omega_r^*

The steady state value of rotor flux

(12) \lambda_{d*} = \frac{2}{\omega_e^*} i_{d*}

The speed dynamic equation

(13) \dot{\omega_e} = \frac{2}{\omega_e^*} (T_e - T_i - B \omega_e)

Substituting (5) in (14) we get

(15) \dot{\omega_e} = \frac{2}{\omega_e^*} (K_t \lambda_{d*} i_{d*} - T_i - B \omega_e)
Assuming the load torque, $T_L$ to be constant, then its derivative will be zero, then the sliding surface for $\omega_r$;

$$S_{\omega_r} = \left( \frac{d}{dt} + \lambda_2 \right) \zeta_{\omega_r}$$

(16)

Where, $n=2$ for the speed.

$$S_{\omega_r} = \dot{\omega}_r - \lambda_2 \omega_r - \zeta_{\omega_r} - \lambda_2 \omega_r$$

(17)

Then,

$$\lambda_2 (\omega_r - \omega_r^*) + S_{\omega_r} = \omega_r - \omega_r^* \tag{18}$$

Where $\lambda_2$ is a positive constant, which determines the band width of the system.

The application of a Lyapunov function candidate is to guarantee the controller stability. Then the choice of the coefficient ($\lambda_2$) will be done in order to keep $\dot{V} = SS \leq 0$

$$\dot{V} = 0 \iff S \leq 0$$ \hspace{1cm} \text{Eq.(19)}

$$V = SS \leq 0$$ \hspace{1cm} \text{Eq.(20)}

Where, $S = \dot{\omega}_r - \lambda_2 \dot{\phi}$

(21)

Then, $\dot{S}_{\omega_r} = \dot{\omega}_r - \lambda_2 \omega_r - \lambda_2 \dot{\omega}_r - \lambda_2 \omega_r$

(22)

Substituting from (18) and (22) in (30)

$$\dot{S} = [\dot{\omega}_r - \lambda_2 \omega_r - \lambda_2 (\omega_r - \omega_r^*)]$$

(23)

From (1) in (23);

$$\left[ (f_0 + \omega_r) (G_2 + U_{\omega_2} - \omega_r^*) \right] \leq 0$$ \hspace{1cm} \text{Eq.(24)}

Where:

$$f_0 = -\frac{e}{J} \omega_r + b \omega_r, b = \frac{L_m}{J} \lambda_2, G_2 = -\frac{e}{J} J \lambda_2$$

Then, $U_{\omega_2} = \lambda_2 \phi V_{\omega_2}$, i.e., the control input.

Equation (24) shows the limits of the coefficient $\lambda_2$.

Get the controller input $U_{\omega_2}$:

Substituting from (1) and (3) in (32) we get:

$$\dot{S} = (G_2 + \lambda_2 \phi - \omega_r^*) + U_{\omega_2} \tag{25}$$

Since, the condition for sliding mode controller is:

$$S_{\omega_r} = 0$$

and then $S_{\omega_r} = 0$

$$U_{\omega_2} = U_{\omega_2} + U_{\omega_1}$$

Then,

$$U_{\omega_2} = (G_2 + \lambda_2 \phi - \omega_r^*) + U_{\omega_1} \text{sgn}(e_{\omega_r})$$

(27)

The second term $U_{\omega_2}$ is the discontinuous control which keeps the system on the sliding surface $S_{\omega_r}$.

Where,

$$\text{sgn}(e_{\omega_r}) = \begin{cases} 1 & \text{if } e_{\omega_r} > 0 \\ -1 & \text{if } e_{\omega_r} \leq 0 \end{cases}$$

The discontinuous control law described by the above equation presents high robustness, insensitive to parameter fluctuations and disturbances.

### 2. Flux Sliding Mode Control

The objective of this controller is making the flux of the machine tracks the reference flux which is a constant value in this case. As the controller designed for speed loop, we can design another one for the flux loop. In this case the reference is $\lambda_{d*}$ and the controller is $V_{d*}$. The error of the flux is defined as:

$$\lambda_{d*} = \lambda_{d*} - \lambda_{d*} \tag{28}$$

Where, $\lambda_{d*}$ is the measured value of rotor flux.

The sliding surface equation:

$$S_{\lambda_d} = \left( \frac{d}{dt} + \lambda_2 \right) \phi \lambda_2$$

(29)

So $S_{\lambda_d} = \left( \frac{d}{dt} + \lambda_2 \right) \phi \lambda_2$, where $n=2$ for the flux.

Then, $S_{\lambda_d} = \phi \lambda_2 \phi_{\lambda_d}$

(30)

$$S_{\lambda_d} = \phi_1 \lambda_2 \phi_{\lambda_d}$$

(31)

$$S_{\lambda_d} = \phi \lambda_2 \phi_{\lambda_d}$$

(32)

Substituting from (1) in (31) and for $\lambda_{d*}$ is constant:

then $\lambda_{d*} = 0$. And $\lambda_{d*} = 0$.

$$S_{\lambda_d} = \lambda_{d*} \phi \lambda_2 \phi_{\lambda_d} - \lambda_{d*} \lambda_{d*} \phi \lambda_2 \phi_{\lambda_d}$$

(33)

$$S_{\lambda_d} = \lambda_{d*} \phi \lambda_2 \phi_{\lambda_d} - \lambda_{d*} \lambda_{d*} \phi \lambda_2 \phi_{\lambda_d}$$

(34)

Where, $f_0 = (a_0 \phi + a_0 \lambda_2)$.

$$G_2 = -\phi \lambda_2 (a_0 \phi + a_0 \lambda_2) + (a_2 \phi + a_2 \lambda_2)$$

and $U_{\lambda_d} = \lambda_{d*} \phi \lambda_2 V_{\lambda_d}$

Equation (34) shows the limits of coefficient $\lambda_2$.

Get the controller input $U_{\lambda_d}$:

From (32):

$$S_{\lambda_d} = \lambda_{d*} \phi \lambda_2 \phi_{\lambda_d} + U_{\lambda_d} \phi \lambda_2$$

(35)

Substituting from equation (1) in (35)

$$S_{\lambda_d} = \lambda_{d*} \phi \lambda_2 \phi_{\lambda_d} + U_{\lambda_d}$$

(36)

Since, the condition for sliding mode controller is:

$$S_{\lambda_d} = \lambda_{d*} \phi \lambda_2 \phi_{\lambda_d} = 0$$

and then $S_{\lambda_d} = \lambda_{d*} \phi \lambda_2 \phi_{\lambda_d} = 0$
\[
U_{ds} = U_{qs} + U_{ns} \quad \text{Then,}
\]
(37)

Where, \( U_{qs} = -\alpha_{2} - \beta_{2}\alpha_{1} + \alpha_{2} \),

And \( U_{ns} = -K_{i} \cdot \text{sgn}(\alpha_{1}) \).

So, the controller input equation for the flux loop:

\[
U_{ds} = [-\alpha_{2} - \beta_{2}\alpha_{1} + \alpha_{2}] - K_{i} \cdot \text{sgn}(\alpha_{1})
\]
(38)

The choice of the two gains \( K_{o}, K_{i} \) must be high enough to guarantee the convergence between reference and measured value. However, this will add more chattering effects to the controller.

3. Reduction of Chattering

As the chattering phenomenon is due to high frequency switching over discontinuity of the control signal (i.e. switching term in the control signal: \( \text{sgn}(e(t)) \)). This effect may cause damages to the controlled physical system.

Therefore, chattering must be reduced or even eliminated of the controller to perform properly. For chattering reduction, several suppression methods have been analyzed recently, including switching gain adaptation methods, the observer-based method and saturation, or use of a shifted sigmoid function instead of a sign function is also used for chattering reduction or elimination. Other chattering suppression methods based on high-order SMC can be found in [26].

In our case, this can be achieved by smoothing out the control discontinuity in a thin boundary layer neighboring the switching surface. The sign nonlinearity can be replaced by another function which can be smoother. There are many types of this function, for example;

\[
\tanh \left( \frac{e}{\alpha} \right)
\]

\( \alpha \) is a small positive constant which reduces the chattering. High value of \( \alpha \) mean more reduction of chattering but more time to reach steady state condition so the optimum value for it must take into account when design the system.

V. SIMULATION RESULTS

In this section, the control scheme is implemented using MATLAB/simulink package program as shown in Fig.2.b. The nominal parameters of the squirrel cage induction motor used for simulation are given in Table I [27].

<table>
<thead>
<tr>
<th>TABLE I. IM Nominal Parameter</th>
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<tbody>
<tr>
<td>Output Power</td>
</tr>
<tr>
<td>Speed</td>
</tr>
<tr>
<td>Line voltage</td>
</tr>
<tr>
<td>Current</td>
</tr>
<tr>
<td>Frequency, f</td>
</tr>
<tr>
<td>Number of pair poles, P</td>
</tr>
<tr>
<td>( R_{o}, R_{r} )</td>
</tr>
<tr>
<td>( L_{s}, L_{r}, L_{m} )</td>
</tr>
<tr>
<td>Moment of inertia, J</td>
</tr>
<tr>
<td>Coefficient of viscous friction, B</td>
</tr>
</tbody>
</table>

The flux command \( \lambda_{d} \) is assumed to be constant and the system is tested for speed tracking. The results are taken first for normal operation with a trapezoidal reference speed and 5 Nm load condition for both PI and SM controllers, Figs (3, 4). Then, some robustness tests are carried out on the machine parameters to measure the robustness of the controller against the parameters variation and the uncertainties in the system model as shown in Figs (5, 6).

The parameters variation will be as follow:

- **Resistance variation**: \( R_{o} \) and \( R_{r} \) will be varied between \( \pm 50\% \) of their normal values.

- **Inductance variation**: \( L_{s} \) and \( L_{r} \) will be changed between \( \pm 20\% \) of their normal values. The change of \( L_{s} \) and \( L_{r} \) by \( +20\% \), and \( -10\% \) instead of \( -20\% \), because during the simulation run, it was remarked that, PI controller mismatch the convergence with the reference values for changes less than \( 14\% \) of the normal value of \( L_{s}, L_{r} \), and need to retuned its gains. For this reason, the comparison is done at \( +20\% \), and \( -10\% \) of \( L_{s}, L_{r} \).
Fig. 3 Speed tracking under nominal parameters and external load application; (a, d) Rotor speed $\omega_r$, rad/sec, (b, e) focus on speed tracking, rad/s, (c, f) Tracking speed error, rad/s.

Fig. 4 Flux tracking under nominal parameters and external load application; (a, d) Rotor flux (Wb), (b, e) focus on the rotor flux, (c, f) d- and q- axis stator currents.
Nominal case analysis:

Figs. 3, 4 show the effect of load application to the IM on both control systems PI and SMC. The speed and flux tracking is excellent in case of SMC, the error in speed is depicted by Fig. 3.e is very small. It is clear that the SMC has a finite-time convergence. On the other side, the convergence time of PI controller is long as shown in Fig. 3.b. From the flux Fig. 4.b, convergence time takes more than 0.3 sec. in case of PI controller, while it takes less than 0.01 sec. in case of SMC as seen in Fig. 4.e. Figs. 3.c, 3.f show the d- and q-axis stator currents. \( i_r \) is constant to maintain the flux command to be constant, and \( i_q \) changes increasingly in both loading and speed change conditions.

Robustness analysis:

Many robustness tests have been run as shown in Figs. 5, 6. Fig. 5 shows the effect of parameter changes on the speed tracking in both PI and SMC. It is obvious that the PI speed controller is sensitive to parameter variation especially rotor parameters \( R_s \) and \( L_s \), as shown in Figs. 5.b, 5.d, and has over shoots in the controller response. Although the change in \( R_s \) Fig. 5.c has a small effect on PI controller but it is visible and stays for a long time. Figs. 5.e – 5.h illustrate the effect of parameter variations on speed tracking using SMC. As can be clearly seen in these figures that the controller is unaffected by the machine uncertainty. The effect of parameters change on flux is displayed in Fig. 6. The PI controller for flux loop is very sensitive to machine parameters as shown in Figs. 6.a – 6.d and this affect on the performance of the controller. While, the SMC for flux loop Figs. 6.e – 6.h show that the system is insensitive to the machine parameter variations. The results which taken to the two controllers proved the efficiency and the robustness of the proposed SM controller compared to PI controller.
VI. CONCLUSION

Linear and nonlinear controllers were presented in this work for a squirrel cage induction motor. A comparison between PI controller and SMC was done. Both controllers were first tested under nominal conditions. Their performances were measured with trapezoidal speed reference and applied external load to the motor.

After that, many robustness tests have been completed successfully. The linear PI controller is not able to achieve good performance, especially against parameter variations. The results proved the efficiency and the robustness of the proposed SM controller.

Since in this paper the motor speed and rotor flux considered as measurable variables, a flux observer and speed estimation method will be implemented in the future work in order to achieve a complete sensorless controller.

References:


