OPTIMAL POWER DISPATCH OF MULTIPLE ENERGY SOURCES IN ENERGY HUBS

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Abstract
Energy consumption is growing in parallel with growth of economy. To meet needs of industrial, commercial and local users and to enable transfer of people and goods, a sufficient supply of energy must be provided. The energy infrastructure needs to be increased and diversified to provide energy needed by each region for future economic growth. The consideration of multiple energy carriers, not only electricity, represents an opportunity for system improvement. The couplings among different infrastructures must be taken into consideration in the light of the idea of “energy hub”. This paper introduces a general modelling and optimization approach for power dispatch and conversion in energy systems including different energy carriers. Loads are supplied using different structures of energy hub to get the optimal structure of the hub. Finally, the approach is demonstrated in numerical case studies.

Index Terms—Energy hub, cogeneration, multiple energy carrier, power dispatch, optimization.

ملخص: يتم استهلاك الطاقة بالتوازي مع نمو الاقتصاد وتلبية احتياجات المستخدمين الصناعيين والتجاريين والمحليين وتتمكن نقل الأشخاص والبضائع، يجب توفير إمدادات كافية من الطاقة تواكب هذا النمو. وذلك تحتاج البنية التحتية للطاقة إلى زيادة مصادرها وتخطيطها لتوفير الطاقة اللازمة لكل منطقة لتحقيق النمو الاقتصادي في المستقبل. وكأحد الحلول الممكنة والعملية يمكن النظر في ناقلات الطاقة المتعددة، وليس فقط الكهرباء، فرصة تحسين أداء وكفاءة النظام. لذلك يجب أن تتخذ في الاعتبار الروابط بين مختلف البنية التحتية في ضوء فكرة "محور الطاقة". تقدم هذه الورقة منهجية للتحسن والتعاون العام لإرسال وتوزيع مصادر الطاقة المختلفة في أنظمة الطاقة. يتم تخفيض الأحمال باستخدام هياكل مختلفة لمحور الطاقة للحصول على الهيكل الأمثل لنقل الأحمال. وأخيراً، تم مناقشة المنهج المقدم في دراسات عدبية مختلفة.
I. INTRODUCTION

arranged and worked dependently [1]. Influenced by energy consumptions growth, researchers suggest integrating these energy systems with different energy carriers rather than concentrating on a single one. This configuration called Energy Hub (EH) [2].

Geidt et al. [3] was the first paper that introduced the model and concept of EH. In [4] some examples of energy hubs are introduced. Ref. [5] presented a complete and basic outline of the most recent models and valuation strategies accessible for the analysis of multi energy systems (MES) and particularly distributed multi-generation (DMG) systems. The studies on EH applications can be classified into two different categories. The first one is the optimal sizing of the energy hub components such as auxiliary boiler, combined heat and power, heating storage and absorption chiller battery [6-9]. In [6] a model to find the optimal size and operation of combined cooling, heat and power was proposed by considering an integrated view of electricity and natural gas network using General Algebraic Modeling System (GAMS) software. An optimal extension arranging model for an energy hub with different energy systems was presented by the authors in [7]. A comparison of two multi-objective optimization forms used to size the elements of an energy hub and to decide their optimal operation due to net present value and carbon emissions was introduced in [8]. Authors in [9] presented an extensive linearized model provided for arrangement

. Ref. [20] proposed an optimization algorithm namely Self-Adaptive-Learning with Time Varying Acceleration Coefficient-Gravitational Search Algorithm (SAL-TVAC-CGSA) to solve energy hub economic dispatch problem. Author in [21] proposed an algorithm which combines optimal dispatch of energy hub, optimal islanding configuration of the system and optimal location of phasor measurement unit (PMU) for a complete control. A modified firefly based algorithm proposed to optimally dispatch the energy hub input energy carriers to minimize the total cost and emission amount of the energy hub [22].

This paper introduces a modeling and optimization method for multi-energy carrier systems in light of the idea of energy hubs. The paper discusses different structures of energy hubs to get the optimal structure of the hub. Different case studies are presented and analyzed, including a base case without any hub, a hub with Combined Heat and Power (CHP) generator, hub with CHP and heat exchanger, and finally a hub with CHP, heat exchanger, and furnace.

The contents of this paper are organized into six sections. After this introduction, energy hub idea is presented. A short overview of the energy hub modelling is presented in section III. Section IV Today's energy systems such as: natural gas, electricity and district heat are mostly and operation of energy hubs considering reliability constraints.

The second category includes different techniques for energy hub optimally control and operation [10-22]. In [10] authors modeled and optimized the energy hub as mixed integer linear problem (MILP). An optimal arrangement of the energy hub considering operation constraints was presented in [11]. In that paper Two Objective Functions (OFs) were represented for deterministic and stochastic circumstances of wind power, electricity price, and the hub electricity demand. Power flow model and optimization method was provided for power systems containing multiple energy carriers in [12]. Ref. [13] proposed an optimization approach to manage the energy hub operation. Ref. [14] proposed a prescient perception technique for enhancing the operation of energy hub systems. Power flow issue in multi carrier energy systems was optimally explained by using additional factors such as dispatch factors [15]. In [16] a disintegration technique was developed and applied to optimal power flow (OPF) of the combined natural gas and electricity. In [17] the power flow issue was optimized for an incorporated arrangement of natural gas and electricity. Ref. [18] focused on reducing the cost of district heating to actively control the supply from the energy hub. In [19] a model of energy hub combined with energy storage devices is proposed and optimal operation technique with renewable energy source (wind) is presented provides detailed formulation of optimization problem. Section V, presents numerical case studies and discusses the obtained results. Finally, section VI concludes the paper.

II. ENERGY HUB CONCEPT

The main idea of energy hub lies in merging different energy carriers and system interaction. Essentially the energy hub is a unit containing output and input ports, conversion devices and storage elements of different energy carriers. Fig.1 presents an example of an energy hub. At the input side, the energy hub is fed by power, natural gas, district heat and wood chips while the output side gives electric power, heating and cooling. The energy carriers at input side are changed over and/or conditioned inside the hub with a specific end goal to provide the load requirements at the output. This hub consists of a furnace and CHP generator as converter elements, a transformer to transfer electrical power from electricity grid to electrical loads, heat exchanger to transfer heat, electrical battery and heat storage as storage elements and absorption cooler to convert heat into cooling.
At the same time, CHP units convert the natural gas to electric power and/or heat. Therefore, electricity and heat which supply the load are affected by such a device. Storage of heated water and electric battery are examples of storage devices that affect the power flow through their integrated work.

Energy hubs can be used in various types of facilities such as industrial, commercial and sometimes residential areas especially in rural and urban regions. There are a number of potential advantages for merging various energy carriers such as increase reliability, load flexibility and optimization potential.

Energy hub concept is so important for many institutions such as:

- Power plants (co- and trigeneration).
- Industrial enterprises (steel factories, paper factories).
- Huge structures (airports, healing centers, shopping centers).
- Adjacent topographical zones (rural and urban regions, towns, urban communities).

### III. MODELLING OF ENERGY HUBS

Power flow inside the hub is described in a mathematical way by a basic steady-state method which is depending on nodal power analysis and just two physical values: power and energy efficiency [18].

Focusing on energy converter’s in- and output power flows, a generic model can be established by assuming the converter as a black box defined by its energy efficiency. A converter may have different in- and outputs, and due to converter’s number of in- and output, four groups of conversion can be defined:

- Single input- single output converter such as: gas furnace which changes natural gas to heat.
- Single input-multiple outputs converter such as: CHP generator with natural gas as an input and electricity, cooling and heating as outputs.
- Multiple inputs-single output converter such as: heat pump with low temperature heat and electricity as inputs and high temperature heat as output.
- Multiple inputs - Multiple outputs converter such as: fuel cell system with hydrogen and oxygen as inputs, electricity and heated water as outputs.

The following subsections present the mathematical models of single input-single output converter (the simplest type) and multiple inputs/outputs converter (the most complicated type).

#### A. Single input single output converter

An energy carrier $\alpha$ at the input is changed into $\beta$ at the output through the converter shown in Fig. 2-a. Power flows at the input and the output are dependent, they can be joined as [1]:

$$ L_{\beta} = C_{\alpha \beta} P_{\alpha} $$

where $P_{\alpha}$ is a steady state energy carrier input, $L_{\beta}$ is a steady state output load and $C_{\alpha \beta}$ is the input/output coupling factor. It is expected that the power flows through the converters are unidirectional, i.e. $P_{\alpha}, L_{\beta} \geq 0$. The converter’s steady-state efficiency mostly identifies the coupling factor. For more precise model, the conversion dependency can be incorporated by expressing the coupling factor as a function of the converted power, i.e., $C_{\alpha \beta} = f(P_{\alpha})$.

$$ P_{\alpha} \xrightarrow{\text{Converter}} L_{\beta} $$

#### B. Multiple input multiple output converter

Using energy carriers and multi-converters inside the energy hub, the coupling matrix $C$ can be defined by different coupling factors [2]:

$$ \begin{bmatrix} L_{\alpha 1} \\ L_{\alpha 2} \\ \vdots \\ L_{\alpha N} \end{bmatrix} = \begin{bmatrix} C_{\alpha 1 \alpha 1} & C_{\alpha 1 \alpha 2} & \cdots & C_{\alpha 1 \alpha M} \\ C_{\alpha 2 \alpha 1} & C_{\alpha 2 \alpha 2} & \cdots & C_{\alpha 2 \alpha M} \\ \vdots & \vdots & \ddots & \vdots \\ C_{\alpha M \alpha 1} & C_{\alpha M \alpha 2} & \cdots & C_{\alpha M \alpha M} \end{bmatrix} \begin{bmatrix} P_{\alpha 1} \\ P_{\alpha 2} \\ \vdots \\ P_{\alpha M} \end{bmatrix} \tag{2} $$

where $P_{\alpha 1}, P_{\alpha 2}, \ldots, P_{\alpha M}$ are the input powers in vector $(P)$, $L_{\alpha 1}, L_{\alpha 2}, \ldots, L_{\alpha M}$ are the outputs in vector $(L)$ as explained by...
The coupling factors are not equivalent to converter’s efficiencies, as long as the converter has more than one input and output. When one energy source is divided into few converters, the coupling factors should be modified by allowed dispatch factors to characterize the input sources of the appropriate converters.

An illustrative example can explain the coupling matrix considerations. For the energy hub shown in Fig. 3, the furnace and CHP generator share the natural gas. Assume that \( \nu \) is the coefficient which is used to determine the distribution ratio of the natural gas where \( 0 \leq \nu \leq 1 \). Then \( \nu P_e \) refer to natural gas input to CHP generator and \( (1 - \nu) P_e \) is the natural gas to the furnace. By and large, each coupling factor is defined by the multiplying the efficiency of the converter and a dispatch factor, i.e. \( C_{\alpha \beta} = \nu \eta_{\alpha \beta} \).

Equation (2) addresses a linear transformation as long as converter’s efficiencies are constant. To get a nonlinear relation, assume the power dependency as \( C(P) = \nu(P) \). Because \( C \) is undefined, \( C \) is non-invertible in either case. This shows the level of flexibility in using input energy carrier to supply loads which are utilized for the optimization. The input-output coupling matrix of an energy hub can be deduced by applying the following steps:

1. Specify output and input power vectors.
2. Detect dispatch factors at input junctions.
3. Define converter output as functions of the inputs.
4. Write nodal power balance at output intersections.
5. Express mathematically the steps from 1 to 4 in (2).

Now, explanation of how to derive the coupling matrix with the help of energy hub shown in Fig. 3. In this example, \( P_e \) and \( P_g \) refer to electricity and natural gas energy carrier at the input and \( L_e \) and \( L_h \) refer to electricity and heat loads at the output.

![Fig. 3: Example of multiple input multiple output energy hub](image)

It is assumed that the converters have constant efficiencies: \( \eta_{ge}^{CHP} \) and \( \eta_{gh}^{CHP} \) for the CHP and \( \eta_{gh} \) for the furnace. For dispatching the aggregate input to the correspondent converter, characterize a dispatch factor \( \nu \) for the natural gas input as explained before.

Converters output can be defined as product of their input and efficiencies after assigning all the parameters:

\[
L_e = P_e + \nu \eta_{ge}^{CHP} P_g
\]

\[
L_h = \nu \eta_{gh}^{CHP} P_g + (1 - \nu) \eta_{gh}^L P_g
\]

\[
\begin{bmatrix}
L_e \\
L_h
\end{bmatrix} =
\begin{bmatrix}
1 & \nu \eta_{ge}^{CHP} \\
0 & \nu \eta_{gh}^{CHP} + (1 - \nu) \eta_{gh}^L
\end{bmatrix}
\begin{bmatrix}
P_e \\
P_g
\end{bmatrix}
\]

With given loads \( L_e \) and \( L_h \), both the input powers \( P_e \) and \( P_g \) as well as the dispatch factors are subject to optimization. After the optimization is performed, the converter inputs can be calculated from the result.

**IV. OPTIMIZATION APPROACH**

Various energy systems at the input side of the hub represent different energy carriers. Through the interior conversion of the hub, the question is how much of each input should be consumed keeping in mind the end goal to optimally meet the load demand (according to the prescribed objective functions, e.g. total cost or \( CO_2 \) emissions). In the hub shown in Fig. 3, there will be many options to supply the loads. All the electric load can be fed from the electricity grid. On other hand, utilizing CHP to convert natural gas to supply a part or whole electric demand, may minimize electricity drawn from the grid and so on. So, optimization goal can be achieved according to the level of flexibility in supplying loads.

**A. Problem Formulation**

The optimization issue can be expressed numerically as follow:

- Define the objective function \( C(P) \) to attain minimum value.
- Derive the coupling matrix \( C \).
- Define the required loads \( L \).
- Define the dispatch factors \( \nu_{\alpha \beta} \) using optimal power inputs \( P \) and the energy carrier dispatch on the converters.
- Define the problem constraints using technical and physical requirements, optimization issues could be compelled.
- Use an optimization tool to find the optimum solution.

The problem can be expressed mathematically as:

\[
\text{Minimize} \quad \text{Cost} (P) = \sum_{\alpha \beta} C_{\alpha \beta} (P_{\alpha \beta})
\]

Subject to

\[
L = C P
\]

\[
P_{\text{min}} \leq P \leq P_{\text{max}}
\]

\[
0 \leq \nu_{\alpha \beta} \leq 1
\]

\[
\sum_{\beta} \nu_{\alpha \beta} = 1
\]

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The inputs are assumed as natural gas, heat, and electrical power from grid. Therefore, the coupling matrix can be deduced using the efficiencies of the converter and the hub topology (as illustrated in Section 3). The coupling matrix can be expressed as:

\[ C = \begin{bmatrix} c_{ee} & c_{ge} & c_{he} \\ c_{eh} & c_{gh} & c_{hh} \end{bmatrix} \]  

(11)

From (11) the minimum value for each energy carrier which supply the loads can be calculated. Substituting (8), (9) and (10) in (12) the cost of each energy carrier can be obtained.

\[ \begin{bmatrix} a_e + 2b_eP_e \\ a_g + 2b_gP_g \\ a_h + 2b_hP_h \end{bmatrix} - \begin{bmatrix} \lambda_e \\ \lambda_g \\ \lambda_h \end{bmatrix} \begin{bmatrix} c_{ee} & c_{ge} & c_{he} \\ c_{eh} & c_{gh} & c_{hh} \end{bmatrix} \begin{bmatrix} P_e \\ P_g \\ P_h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]  

(12)

From (12) the total benefit of energy carriers at the output of the hub can be calculated.

**Total benefit** = \( \lambda_e + \lambda_h \)

(13)

C. **Optimization procedure**

For simple case studies, the optimization problems can be solved using the first order KKT as illustrated before. But in more complex cases, Artificial Intelligence tools must be used to save time and to efficiently reach to an optimal solution. In this study, genetic algorithm (GA) was used to solve complex problems because GA is so powerful that they can exhibit more efficiency if programmed perfectly. The optimization procedure using GA can be summarized in the following steps:

1. Read hub data, load value, coupling matrix, and converter efficiencies.
2. Use optimization technique to determine input power from each supply, and dispatch factors \( \upsilon_{\alpha,i} \).
3. Check the limitation of the constraints.
4. If the solution is the best, end, else go to step 2

V. **CASE STUDIES AND RESULTS**

To clarify the optimization approach, different structures of energy hubs are studied. The studied hubs are composed of different loads such as heat and power supplied by different inputs such as electricity, natural gas and heat. Loads are supplied using different structures of energy hub to get the optimal hub structure. Four case studies are discussed including a base case without any hub, a hub with CHP generator, hub with CHP and heat exchanger, and finally a hub with CHP, heat exchanger, and furnace.

In each case study, the loads to be fed are an electric load, \( L_e = 50 \) kW and a heat load, \( L_h = 150 \) kWh. The efficiencies of the CHP generator are assumed to be constant values \( \eta_{CHP_e} = 0.35 \) (gas-electricity) and \( \eta_{CHP_h} = 0.4 \) (gas-electricity). The efficiencies of the heat exchanger and the furnace are \( \eta_{HE} = 0.9 \) and \( \eta_f = 0.75 \) respectively. The cost
coefficients for every energy carrier at the hub input is taken as given in Table 1.

<table>
<thead>
<tr>
<th>Carrier</th>
<th>( a_i ) ($/kW$)</th>
<th>( b_i ) ($/kW^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity (e)</td>
<td>12</td>
<td>0.12</td>
</tr>
<tr>
<td>Natural gas (g)</td>
<td>5</td>
<td>0.05</td>
</tr>
<tr>
<td>Heat (h)</td>
<td>4</td>
<td>0.04</td>
</tr>
</tbody>
</table>

A. Case Study 1 (no energy hub)

In the absence of energy hub, each load is fed from one supply only. Applying (11)

\[
\begin{bmatrix} 50 \\ 150 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_e \\ P_g \\ P_h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

The unique solution is given as \( P_e = 50 \text{ kW}, P_g = 0 \) and \( P_h = 150 \text{ kWth} \) (the same value of the required loads). The total cost in this case is 40 $/kW.

B. Case Study 2 (with CHP generator)

In this case, the studied hub supplies two types of load electrical and thermal loads and is fed by three inputs: natural gas, electricity and heat. The hub contains a CHP generator which converts natural gas into electricity and heat. Electric load can be fed directly from the electricity grid or indirectly through the CHP generator. Heating load can either be fed specifically from the district heat or indirectly through the CHP generator as explained by Fig. 4.

\[
\begin{bmatrix} 50 \\ 0 \\ 150 \end{bmatrix} - \begin{bmatrix} 1 & 0.35 & 0 \\ 0 & 0 & 0 \\ 0 & 0.40 & 1 \end{bmatrix} \begin{bmatrix} P_e \\ P_g \\ P_h \end{bmatrix} = \begin{bmatrix} 28.8499 \\ 60.4288 \\ 125.8285 \end{bmatrix}
\]

Assuming direct lossless connections in the electrical and thermal network, so the system's marginal price (SMP) and load marginal price (LMP) are equal. The total cost in this case is 32.2844 $/kW.

In this case, the use of optimally operated CHP generator decreases the total energy cost by 19% compared to the first case study.

C. Case study 3 (with CHP and heat exchanger)

The studied hub consists of CHP generator and a heat exchanger with the same three inputs and two outputs as explained before. Again, the electric load can be fed directly from the electricity grid or indirectly through the CHP generator. Heating load can be fed from the heating network through the heat exchanger and/or from the CHP generator as illustrated by Fig. 5.

\[
\begin{bmatrix} 50 \\ 0 \\ 150 \end{bmatrix} = \begin{bmatrix} 1 & 0.35 & 0 \\ 0 & 0 & 0 \\ 0 & 0.40 & 0.9 \end{bmatrix} \begin{bmatrix} 25.7196 \\ 69.3727 \\ 135.8344 \end{bmatrix}
\]

The electrical SMP and LMP are equivalent since assuming that there is a direct lossless connection to the network. The total cost in this case is 34.032 $/kW.

In this case, optimally operation of CHP and heat exchanger reduces the total energy cost by 14% in comparison with the first case study. On other hand, the total cost is increased by 5% compared with the second case study due to the presence of heat exchanger.

D. Case study 4 (with CHP, heat exchanger, and furnace)

The studied hub consists of CHP generator a heat exchanger and a furnace which converts natural gas into heat with the same three inputs and two outputs as explained before. In this hub, the heating load can be fed by the CHP generator the furnace or the heat exchanger as illustrated by Fig. 6. In this case the natural gas is divided between the CHP and the furnace by a certain percentage \( \nu \). In this paper, the optimum value of \( \nu \) is obtained using GA as explained by Fig. 7.
The input power flow and marginal price couplings can be expressed using (11) and (8) as:

\[
\begin{bmatrix}
50 \\
0 \\
150
\end{bmatrix}
= \begin{bmatrix}
1 & 0.35v & 0 \\
0 & 0 & 0 \\
0 & 0.4v + 0.75(1 - v) & 0.9
\end{bmatrix}
\begin{bmatrix}
25.7196 \\
69.3727 \\
135.8344
\end{bmatrix}
\]

As stated before, both the SMP and LMP are equivalent since there is direct lossless connection to the network. The total cost in this case is 34.032 $/kW.

In this case the results show that the optimal value of \( v \) is equal 1 which means that all natural gas will utilized by the CHP and hence there is no effect of the furnace on the optimization process, so that the minimum cost is the same as the previous case study.

From table 2, it is clear that case study no. 2 is the optimal structure of the hub that supplies the loads at minimal cost.

From Fig. 8, it is shown that case 1 require the biggest value of electricity and heat from the grid, which decrease in the other cases while natural gas value appear. Case no. 2 require the minimum value of all energy carriers because the hub in this case contains only CHP while cases 3 and 4 contain more devices which cause more losses. Case 3 and 4 require the same value of energy carriers because the furnace has no effect on the optimization process as explained before.

![Fig. 6. A hub containing CHP, heat exchanger and furnace](image)

![Fig. 7. Minimum cost of case study 4](image)

![Fig. 8. Comparison of each carrier](image)
VI. CONCLUSION

This paper demonstrated the idea of energy hubs which can be considered as helpful as a next step to co-and trigeneration systems through a series of examples. The paper introduced a general modelling and optimization approach for power dispatch and conversion in energy systems including different energy carriers. Different structures of energy hubs were studied to get the optimal structure of the hub. Four case studies were discussed including a base case without any hub, a hub with CHP generator, hub with CHP and heat exchanger, and finally a hub with CHP, heat exchanger, and furnace.

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