Real-Time Harmonic Reduction using Synchronous PWM Control for Wound-Rotor Induction Motor

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Abstract In this paper the dynamic performance of the wound rotor induction motor operating with synchronized modulator is considered and analyzed. The proposed modulator employs a PWM transistor-controlled capacitive network in rotor circuit with a carrier frequency proportional to the rotor voltage frequency. This modulator can reduce the harmonic current components of the motor and consequently improve the motor power factor. The reduction of these components is achieved instantaneously with no need for sensing or computing the harmonics current in motor current, thus simplifying the control system. Simulated and experimental results obtained from closed loop 0.5KVA prototype confirm the feasibility and features of the proposed system.

1. Introduction
Adjustable speed control of induction motor (IM) via static power converter is increasingly based on real time digital generation of PWM waveforms. Different strategies for optimizing PWM for voltage source inverter in the stator side of IM such as synchronous, asynchronous PWM and regular sampling have been proposed and analyzed [1-8]. However, controlling the speed of wound-rotor IM can be achieved by using external variable rotor resistance or by resonating the rotor circuit using uncontrolled reactive rotor network [9]. These systems provide high starting and braking torque, improved power factor and also reacts favorably to non-sinusoidal supply voltage. For smooth variation of motor speed, the reactive rotor networks have been controlled using thyristor switches [10-12]. Also, transistor switches strategy has been proposed using non-optimal PWM techniques [13-14]. These switches lead to significant increase of harmonic distortion in machine currents. Such high power PWM electronic switches are generally operated at low-switching frequency, owing to limitation of the semiconductor switches. This needs an optimization for the PWM switching sequences, aiming at a reduction of harmonic components of the machine currents, and a reduction in torque harmonics.

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of the capacitive rotor network to control the speed of the wound rotor
induction motor. This synchronized modulator is extended over a full
fundamental period of the rotor voltage so that the switching of the capacitive
rotor network is in synchronism with the rotor voltage frequency. The
resulting pulse sequence exhibits quarter wave symmetry. Synchronized pulse
sequences are advantageous in that the harmonic spectrum does not contain
sub-harmonic components and minimize the lower order harmonic distortion on
the machine currents.

2. System Description

2-1. Power circuit

The schematic for the proposed synchronous PWM control for wound rotor IM
is shown in Fig 1. Three capacitors are star connected to the rotor via a three-
phase power controller using four IGBT switches. T1 and T3 are shunt switches
while T2 and T4 are series switches. Each transistor is fed through four fast
recovery diode bridge to act as ac switch as shown in Fig 1.

Fig. 1. Schematic diagram of the PWM with a synchronized carrier wave.

The switching process is to disconnect the capacitor (series switches are OFF
while shunt switches are ON) or connect these capacitors (shunt switches are
OFF while series switches are ON) in the rotor circuit. The motor rotates at its
rated speed when shunt switches are continuously ON. While when the series
switches are continuously ON, the capacitors are permanently connected to the
rotor. To vary the motor speed over the whole range, the value of the
capacitors is chosen by varying the duty cycle of the series switches from zero
to 100% (i.e. from 100% to zero of the shunt switches), motor speed changes
from its rated value to zero.

2-2. Control circuit

The control circuit of the system can be subdivided into synchronized
modulator (for open and closed loop) and PI controller for closed loop speed
regulation as shown in Fig 1. The synchronized modulator consists of a triangular carrier signal generator with output frequency $f_s$ which is synchronized with rotor voltage frequency $f_r$ by a fixed frequency ratio $N$. PWM comparator compares the synchronized-frequency carrier signal with a modulating signal $v_m$ (open or closed loop). The comparator provides a train of synchronized pulses with a duty cycle inversely proportional to $v_m$ and its frequency is the carrier frequency. The output of synchronized modulator is fed to the drive circuit through a NAND latch circuit. This circuit complements the synchronized pulses and ensures that, any group of series or shunt switches turned OFF just after the others are ON. The relative duty cycle of the series switches determines capacitor voltage level relative to that of the rotor and hence achieving the desired motor speed. The capacitor voltage increases due to increasing the duty cycle of series switches by reducing $v_m$ (open loop) or $\omega_{\text{ref}}$ (closed loop).

PI controller circuit for control of a power transistor, which operate in a high frequency chopping mode, such that the duty cycle can be smoothly controlled the motor speed $\omega_m$ to the desired speed $\omega_{\text{ref}}$ in the range 0 to 100%.

3. Synchronized Modulator

Two mathematical models have been proposed for modeling the switched-control through the rotor of the induction motor [15,16]. These models are only restricted in time domain analysis. The frequency domain analysis will be used to study the motor behaviour with the proposed synchronized modulator.

In the present study, the structure considered in Fig 1, is open loop feed forward system. Effectively, the capacitors can be considered as a voltage source $v_c$ connected to the rotor circuit and has the same rotor voltage frequency. Accordingly, the capacitor voltage $v_c$ is sinusoidal. The switch-on duration $T_{on}$, Fig 2, of $v_c$ across the rotor circuit is $v_a$. Due to high switching frequency, the switched voltage $v_a$ can be approximated by a sequence of flattopped pulses with sinusoidal shape amplitude of $v_c$, since the variation of $v_c(t)$ in the switching duration will not be significant. Then, $v_a(t)$ can be written as,

$$v_a(t) = \left\{ \begin{array}{ll}
v_c \quad \text{for} \quad kT_s \leq t < kT_s + T_{on} \\
0 \quad \text{for} \quad kT_s + T_{on} \leq t < (k+1)T_s \\
\end{array} \right.$$

where, $k = 0, 1, 2, \ldots$

$T_s$ is the period of switching frequency $f_s$.

The switched voltage $v_a^*(t)$ can be expressed as an infinite series,

$$v_a^*(t) = \sum_{k=0}^{\infty} v_c(kT_s)[u_s(t-kT_s) - u_s(t-kT_s - T_{on})] \quad (1)$$

where $u_s(t)$ is the unit-step function.

Taking the Laplace transform, $S$-domain, on both sides of eq (1), it follows that,
The term $e^{-T_{on}S}$ can be approximated by taking only the first two terms of its power-series expansion, then,

$$1 - e^{-S T_{on}} = 1 - [1 - S T_{on} + \frac{(S T_{on})^2}{2!} + \cdots] \approx S T_{on}$$

Thus, eq (2) is simplified to,

$$V_a^*(S) \approx T_{on} \sum_{k=0}^{\infty} v_c(k T_s) e^{-k S T_s} = T_{on} \sum_{k=0}^{\infty} v_c(k T_s) z^{-k}$$

In the Z-domain, eq (3) is equivalent to,

$$V_a(z) \approx T_{on} V_c(z)$$

For the sinusoidal waveform, the Z-transform of $v_c(t)$ is [17],

$$V_c(z) = \frac{z \sin \omega_r T_s}{z^2 - 2 z \cos \omega_r T_s + 1}$$

Accordingly, eq (4) becomes,

$$V_a(z) \approx T_{on} \frac{z \sin \omega_r T_s}{z^2 - 2 z \cos \omega_r T_s + 1}$$

$$\approx T_{on} \frac{e^{S T_s} \sin \omega_r T_s}{e^{2 S T_s} - 2 e^{S T_s} \cos \omega_r T_s + 1}$$

where $\omega_r = 2 \pi f_r$ and $f_r$ is the rotor voltage frequency.

For steady state $S = j \omega_r$ and $T_s = 1 / f_s$ then eq (5) becomes,

$$V_a(z) \approx T_{on} \frac{e^{j 2 \pi f_r / f_s} \sin 2 \pi \frac{f_r}{f_s}}{e^{j 4 \pi f_r / f_s} - 2 e^{j 2 \pi f_r / f_s} \cos 2 \pi \frac{f_r}{f_s} + 1}$$

For fixed-frequency carrier, varying $v_m$ will vary each of motor speed $\omega_m$, rotor voltage frequency $f_r$, and switch-on duration $T_{on}$. Accordingly, both the phase and magnitude of $V_a(z)$ will be varied. This leads to obtaining variable harmonic current components for each motor speed.

For the proposed modulator, with a fixed frequency ratio $N = f_s / f_r$, eq (6) becomes,

$$V_a(z) \approx T_{on} \frac{e^{j 2 \pi / N} \sin 2 \pi / N}{e^{j 4 \pi / N} - 2 e^{j 2 \pi / N} \cos 2 \pi / N + 1}$$

The phase and magnitude of the division term in eq (7) become fixed with speed variation. However, the variation of $T_{on}$ with motor speed will be studied as given below.
Figure 3, shows the speed variation versus the reference voltage $v_m$ at certain capacitance ($c=14\mu F$) for fixed and synchronized-frequency carriers with constant load torque. Such variations was measured primarily to evaluate the behaviour of motor speed with $v_m$. It is noticed that the motor speed is linearly varied with $v_m$ i.e. $\omega_m = k' (v_m - 0.4)$ (8)

Accordingly, and from Fig 2 using triangles uniformity rule,

$$\frac{T_{on1}}{T_{sl}} = f_{sl} = 1 - \frac{v_{ml}}{V_{m max}}$$ (9)

since,

$$f_{sl} = N \frac{\omega - \omega_{m1}}{\omega} = N(1 - \frac{\omega_{m1}}{\omega})$$ (10)

where, the slip 's' is given by, $s = (\omega - \omega_{m1})/\omega$, and $f$ is the supply frequency.

From eq (8) and by considering the synchronous motor speed $\omega = k' (V_{m max} - 0.4)$ for simplicity, eq (10) becomes,

$$f_{sl} = \frac{N}{\xi} (1 - \frac{v_{ml}}{V_{m max}})$$ (11)

and eq (9) becomes,

$$\frac{N}{\xi} (1 - \frac{v_{ml}}{V_{m max}}) f T_{on1} = 1 - \frac{v_{ml}}{V_{m max}}$$ (12)

where, $\xi = 1 - \frac{0.4}{V_{m max}}$

For a step change of speed reference voltage to $V_{m2}$ at steady state, eq (12) becomes,

$$\frac{N}{\xi} (1 - \frac{v_{m2}}{V_{m max}}) f T_{on2} = 1 - \frac{v_{m2}}{V_{m max}}$$ (13)

Dividing eq (12) by eq (13) gives,

$$T_{on1} = T_{on2} = \xi / (N f)$$ (14)

This means that, for the variation of speed reference voltage $v_m$ with constant load torque, the switch-on duration $T_{on}$ remains unchanged, while the relative duty cycle vary to ensure capacitor voltage required to achieve the desired
With this result and for constant frequency ratio \( N \), the phase and magnitude of eq (7) remains unchanged for wide range of motor speed.

So, the purpose of the proposed modulator is,

- to maintain the number of switching pulses for the rotor voltage always fixed, i.e. the switching frequency \( f_s \) is in synchronism with the fundamental frequency of the rotor voltage \( f_r \) by the frequency ratio \( N \). The resulting pulse sequence exhibits quarter wave symmetry such that the harmonic spectrum of the rotor voltage does not contain sub-harmonic current component, especially at lower switching frequencies.

- to change the naturally variable phase and magnitude of the switched capacitor voltage \( v_a \), eq (6), to fixed phase and magnitude, eq (7) for wide range of motor speed. With this property of fixed phase and magnitude, the effect of the capacitor \( C \), which can be optimally chosen at a certain speed and switching frequency to give minimum low order harmonic currents, remains valid for a wide range of motor speed.

4. Mathematical Model

Considering the per-phase equivalent circuit of the capacitor controlled IM referred to the stator shown in Fig 4, the following motor equations can be written,

\[
v_s = i_s r_s + \ell_s (d i_s / dt) + i_m r_m
\]

\[
i_m r_m = \ell_m (d i_m / dt)
\]

\[
i'_r = i_s - (\ell_m / r_m) (d i_m / dt) - i_m
\]

\[
i_m r_m = i'_r r_m / s + \ell'_r (d i'_r / dt) + v_a
\]

\[
v'_a = 0 \quad \text{if } T_a \text{ ON and } T_b \text{ OFF} \quad (19)
\]

Hence,

\[
v'_a = v'_c / s,
\]

\[
i'_r = s^2 c' (d v'_a / dt) \quad \text{if } T_a \text{ OFF and } T_b \text{ ON} \quad (20)
\]

The developed motor torque is given by,

\[
T_m = 3 (I_r^2 / \omega_m) \quad (21)
\]

The electromechanical equation is given by,

\[
J (d \omega_m / dt) = T_m - T_L - \beta \omega_m \quad (22)
\]

Equations (15) to (22) have been used in ref (13) to calculate the motor performance under the condition of fixed-frequency carrier. In the present study the same equations are reprogrammed along with the following equations to simulate the system with the proposed modulator. The triangle carrier signal \( E_c(t) \) of the synchronized PWM can be expressed by Fourier analysis as,

\[
e_c(t) = [1 + \frac{8}{\pi^2} \sum_{n=1,3,5,...}^{\infty} \frac{1}{n^2} (-1)^{(n-1)} \sin n \omega_s t] E_c / 2 \quad (23)
\]

Where \( E_c \) is the peak value of the triangular carrier signal and \( \omega_s = 2\pi N s f \) is its angular frequency. The maximum value \( V_{\max} \) of the modulation degree is adjusted to be equal to \( E_c \) to provide modulation degree \( M = V_{\max}/E_c = 1 \).
Comparing $e_c(t)$ with $v_m(t)$ yields the switching pattern and PWM comparator $V_{PWM}$ as,

$$V_{PWM} = V_{cc} \text{ for } e_c(t) > v_m(t) > 0 \quad T_a \text{ OFF and } T_b \text{ ON}$$

$$= 0 \text{ for } e_c(t) < v_m(t) > 0 \quad T_a \text{ ON and } T_b \text{ OFF}$$

Generally, with synchronized PWM modulator, $V_{PWM}(t)$ will be a periodical function with the fundamental frequency.

![Figure 3. Measured motor speed $\omega_m$ variation with the speed reference voltage $v_m$.](image)

![Fig. 4. Per phase motor equivalent circuit.](image)

5. Experimental Setup

The experimental setup is illustrated in Fig 1 and was developed to test the proposed synchronized modulator for controlled wound rotor IM. The synchronized modulator is a very simple replacement of the conventional analog triangle regulator by triangular generator of FET type. The shunt ($T_1$ and $T_3$) and series ($T_2$ and $T_4$) switches are IGBT switches of type 25Q101 ($V_{CEO}=1500$ V, $I_C=50$A). An analog current controlled separately excited dc motor provides the required load torque. Parameters of employed induction and dc machines are listed in the Appendix. A PC Pentium is used to program the Lab-pc 1200 I/O card using the LabView software to display experimental data. The interface between the I/O card and the system was done via a Hall effect sensors (LA25 and LV25 from LEM) for measuring the stator and rotor phase currents and voltages, respectively. A tacho-generator with a gain $2V/1000rpm$ is employed for measuring the instantaneous motor speed as well as for the feedback signal. The tacho-generator output is directly connected to the I/O card. The parameters for the PC controller ($k_p + k_i/s$) were adjusted empirically by means of computer simulation for a certain operating point (at 66% of rated speed =1000r.p.m). The constant $k_p$ for proportional controller was adjusted to 20, and the integrator $k_i=80s^{-1}$ was used to obtain 5% overshoot of motor speed within 100 m-sec.

6. Simulation and Experimental Results

Several computer simulations and experiments were run with different operating points in order to check the performance of synchronized modulator with closed loop system. Equations (15) to (25) listed above were solved using software Matlab-Simulink to obtain the motor performance under the proposed
modulator. Figures 5 and 6 show the simulated motor speed and current waveforms at the same conditions without and with the synchronized modulator, respectively. These results were taken using PI controller at different step reference voltages at 0.75 N-m load torque. The motor speeds follow the reference voltages and the steady state error for each reference is zero as shown in Figs 5a and 6a. Figure 6a ensures that the synchronized modulator makes the motor speed arrive rapidly to the reference than in Fig 5a.

Figures 5b and 6b show expanded waveforms of the applied voltage $v_s$, stator current $i_s$ and referred rotor current $i_r$ before the sudden change of the speed reference $\omega_{ref}$ from 1200 to 800rpm. It is seen from these figures that the currents $i_s$ and $i_r$ are sinusoidal due to the used synchronized modulator.

Figures 5c and 6c show expanded waveforms of the applied voltage $v_s$, stator current $i_s$ and referred rotor current $i_r$ before the sudden change of the speed reference $\omega_{ref}$ from 1200 to 800rpm. It is seen from these figures that the currents $i_s$ and $i_r$ are sinusoidal due to the used synchronized modulator.

Fig. 5. Simulation results for closed loop motor control with fixed-frequency switching.

Fig. 6. Simulation results for closed loop motor control with synchronized-frequency switching.

Linear scales plot of $I_r$ and $I_s$ spectra without and with the synchronized modulator are shown in Figs 5c and 6c, respectively. The results clarify the effect of the synchronized modulator in reducing the motor current harmonics produced by the fixed-frequency switching.

Figures 7 and 8 show the experimental waveforms of $v_s$, $i_s$, $v_s$, and $i_s$ at closed loop motor control for a sudden change of speed reference $\omega_{ref}$ and constant load torque. In Fig 7 the speed changes from 1200 to 1040rpm with fixed switching frequency $f_s$ is 900Hz. This low switching frequency is chosen at 1200 rpm, i.e. $f_r$ is 10Hz, and the stator voltage $v_s$ is reduced to 190V to have
significant current harmonics and small-sustained oscillations in the motor current waveform as shown in Figs 7b and 7d. These harmonics and oscillations are increased dramatically when the speed is decreased to 1000rpm, i.e. $f_r$ increased to 16.667Hz while the switching frequency remains without change at 900Hz.

![Stator voltage $v_s$](image1)

![Stator current $i_s$](image2)

![Switched voltage $v_s$](image3)

![Rotor current $i_r$](image4)

![Expanded $v_s$ and $i_s$ for $\omega_m=1200$ rpm](image5)

![Expanded $v_s$ and $i_s$ for $\omega_m=1040$ rpm](image6)

![Expanded $v_s$ and $i_s$ for $\omega_m=1000$ rpm](image7)

**Fig. 7.** Experimental results for closed loop motor control with fixed-frequency switching for speed reference variations at constant load and $V_s=190V$.

![Stator voltage $v_s$](image8)

![Stator current $i_s$](image9)

![Switched voltage $v_s$](image10)

![Rotor current $i_r$](image11)

![Expanded $v_s$ and $i_s$ for $\omega_m=1200$ rpm](image12)

![Expanded $v_s$ and $i_s$ for $\omega_m=1040$ rpm](image13)

![Expanded $v_s$ and $i_s$ for $\omega_m=1000$ rpm](image14)

**Fig. 8.** Experimental results for closed loop motor control with synchronized-frequency switching for speed reference variations at constant load and $V_s=220V$.  

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In Fig 8, the synchronized modulator with fixed frequency ratio N of 90 is used. This ratio can be changed to the values with only a small adjustment in triangle generator shown in Fig 1. Figure 8 shows the system response, since the motor speed is varied from 1200 to 1000rpm with \( v \) 220V. Accordingly, \( f_r \) is varied from 10 to 16.667 Hz, consequently the synchronized modulator adapt \( f_r \) from 900 to 1500Hz respectively. As expected from the analysis of the proposed modulator and referring to eq (7), where the phase and magnitude of \( v_a \) remains unchanged, the motor current waveforms remain sinusoidal in the steady state for both speeds of 1200 and 1000rpm. This is true except with high switching ripple super-imposed on the motor currents.

Figure 9 shows the experimental waveforms of \( i_{\text{ref}}, \omega_{\text{ref}}, v_a, \) and \( i_r \) for closed loop motor control with synchronized switching frequency with step change of load torque at constant reference speed. Figures 9e to 9h show that, the steady state motor currents remain nearly sinusoidal before and after loading. These results according to eq (7) where only the phase of \( v_a \) remains unchanged while its magnitude is continuous variation due to the dependency of \( T_{\text{on}} \) on load value. This leads to the nearly sinusoidal appearance.

![Graphs of experimental results](image)

Fig. 9. Experimental results for closed loop motor control with synchronized switching frequency for load variation with constant speed reference at \( V_r=220V \).

7. Conclusions

The present paper deals with the implementation of a synchronized PWM in conjunction with closed-loop control of wound rotor IM. The synchronized modulator has employed a PWM transistor-controlled capacitive network in rotor circuit with a carrier frequency proportional to the rotor voltage frequency. It has been shown that, the modulator maintain the number of switching pulses for the rotor voltage always fixed which exhibit quarter wave...
symmetry and reduce the sub-harmonic current component, especially at lower switching frequency. The mathematical analysis shows that, the naturally variable phase and magnitude of the switched capacitor voltage has been changed to fixed phase and magnitude for wide range of motor speed. With this property, the effect of the rotor capacitor, which can be optimally chosen at a certain speed and switching frequency to give minimum low order harmonic currents, remains valid for a wide range of motor speed. A power factor improvement is necessarily obtained by this harmonic content reduction. Simulations and experimental results have been shown to be in good agreement with the mathematical analysis.

The proposed modulator is also valid for the PWM inverter feeding the stator of the ac drives recently developed with vector control, direct torque, and space vector techniques. These techniques have the same drawbacks of the system under consideration when using a fixed-frequency switching that affects the drive performance.

References


Appendix

- Specification of tested induction motor: $V_s=220 $ V, $2p=4$, $f=50$ Hz, $n_m=1340$ rpm, $I_s=1.16$ A.

Appropriate calculations of no load, locked rotor tests, and open circuit tests gave the following results:

- $r_p=35$ $\Omega$, $\ell_s=0.17$ $\Omega$, $r_r=2.1$ $\Omega$
- $\ell_s=0.0106$ $\Omega$, $r_m=3400$ $\Omega$, $\ell_m=0.99$ $\Omega$

Primary / secondary transformation ratio=4

$\beta=0.00075$ N-m /rad/ sec.

$I$ (motor-load inertia)=0.00035 N. m /rad / sec$^2$.

- Specifications of separately excited dc machine: 220V, 1.2A, 1500rpm and field voltage is 220v.
تقليل التوافقيات الناتجة عن التحكم في سرعة المحركات التأثيرية ذات العضو الدائري المكلف باستخدام معدل نبضات تذامني

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ملخص البحث

تعد دوار القوة الإلكترونية المصدر الرئيسي للتوليفيات في أنظمة التسيير الكهربائي. ويقدم هذا البحث دراسة مفصلة وعملية للطريقة الجديدة لتقليل التوليفيات ضرر نتائج يبت في التحكم في سرعة مجموع مكلفات مفصلة مع بعضها الدوار باستخدام حاكم تذامني - تكامي. وتعد طريقة التحكم هذه على استخدام مكلفات مساوية السعة متصلاً بجهاز في دائرة العضو الدوار يبت التحكم فيها عن طريق أربعة مفاتيح إلكترونية سريعة يمكنها تحويل النياز في كل الأحجام باستخدام فترات من أربعة موحدات سريعة لكل مساح. ويمكن التحكم بعمود في السرعة بالتحكم في فترات اتصال المكلفات بدائرة العضو الدوار بالنسبة للساعة استيعابها وقصير دائرة العضو الدوار وبالتالي يمكن التحكم في جهد المكلفات والذي يعمل كمصادر جهد ثلاثية الأوجه مضاد جهد العضو الدوار ولد نفس تردد ويدخل المكلفات أو إعادةها في دائرة العضو الدوار بأسلوب التعديل التذامي لجهاز المكلفات.

وقد أثبتت الدراسة أنه عند استخدام تردد ثابت ل معدل النبضات أن قيمة وروازية جهد المكلفات المقطع على دائرة العضو الدوار تتغير مع تغير تردد جهد العضو الدوار بسبب دورة توليفيات متعددة في تيارات المحرك. وفي هذا البحث تم افتراض طريقة تشير تردد معدل النبضات ليكون متسامياً مع تردد جهد العضو الدوار بحيث تخفف دورة جهد المكلفات المقطع على دائرة العضو الدوار دائماً بعدد ثابت من النبضات مع تغير سرعة المحرك. أثبتت الدراسة النظرية للطريقة المقررة أن معدل النبضات المتزامن يعمل على تثبيت قيمة ورازوية جهد المكلفات المقطع عند تغير سرعة المحرك. ويوجد هذه الحفاظة - لمدى واسع سرعات - على أقل قيمة توليفيات تيار المحرك تصل إلى قيمة مقصودة لكل من سعة المكلفة وتتردد التقلص وسرعة المحرك.

تم تحليل النظام وبناء النموذج النظري حيث ظهر توليفاً جيداً بين النتائج النظرية والعملية وأوضحت كل من النتائج العملية والعائية نظرية أن استخدام هذه الطريقة يؤدي إلى تقليل التوليفيات في تيار المحرك وتحسين معامل القدرة مقارنة باستخدام معدل نبضات ذو تردد ثابت.